

Useful Inequalities $\{x^2 \geq 0\}$

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Cauchy-Schwarz	$\left(\sum_{i=1}^n x_i y_i\right)^2 \leq \left(\sum_{i=1}^n x_i^2\right) \left(\sum_{i=1}^n y_i^2\right)$		
Minkowski	$\left(\sum_{i=1}^n x_i + y_i ^p\right)^{\frac{1}{p}} \leq \left(\sum_{i=1}^n x_i ^p\right)^{\frac{1}{p}} + \left(\sum_{i=1}^n y_i ^p\right)^{\frac{1}{p}} \quad \text{for } p \geq 1.$		
Hölder	$\sum_{i=1}^n x_i y_i \leq \left(\sum_{i=1}^n x_i ^p\right)^{1/p} \left(\sum_{i=1}^n y_i ^q\right)^{1/q} \quad \text{for } p, q > 1, \frac{1}{p} + \frac{1}{q} = 1.$		
Bernoulli	$(1+x)^r \geq 1+rx \quad \text{for } x \geq -1, r \in \mathbb{R} \setminus (0,1). \text{ Reverse for } r \in [0,1].$ $(1+x)^r \leq \frac{1}{1-rx} \quad \text{for } x \in [-1, \frac{1}{r}), \quad r \geq 0.$ $(1+x)^r \leq 1 + \frac{x}{x+1} r \quad \text{for } x \geq 0, \quad r \in [-1, 0].$ $(1+x)^r \leq 1 + (2^r - 1)x \quad \text{for } x \in [0, 1], \quad r \in \mathbb{R} \setminus (0,1).$ $(1+nx)^{n+1} \geq (1+(n+1)x)^n \quad \text{for } x \in \mathbb{R}, n \in \mathbb{N}.$ $(a+b)^n \leq a^n + nb(a+b)^{n-1} \quad \text{for } a, b \geq 0, n \in \mathbb{N}.$		
exponential	$e^x \geq \left(1 + \frac{x}{n}\right)^n \geq 1 + x; \quad \left(1 + \frac{x}{n}\right)^n \geq e^x \left(1 - \frac{x^2}{n}\right) \quad \text{for } n \geq 1, x \leq n.$ $\frac{x^n}{n!} + 1 \leq e^x \leq \left(1 + \frac{x}{n}\right)^{n+x/2}; \quad e^x \geq \left(\frac{ex}{n}\right)^n \quad \text{for } x, n > 0.$ $x^y + y^x > 1; \quad x^y > \frac{x}{x+y}; \quad e^x > \left(1 + \frac{x}{y}\right)^y > e^{\frac{xy}{x+y}} \quad \text{for } x, y > 0.$ $\frac{1}{2-x} < x^x < x^2 - x + 1; \quad e^{2x} \leq \frac{1+x}{1-x} \quad \text{for } x \in (0, 1).$ $x^{1/r}(x-1) \leq rx(x^{1/r}-1) \quad \text{for } x, r \geq 1; \quad 2^{-x} \leq 1 - \frac{x}{2} \quad \text{for } x \in [0, 1].$ $xe^x \geq x + x^2 + \frac{x^3}{2}; \quad e^x \leq x + e^{x^2}; \quad e^x + e^{-x} \leq 2e^{x^2/2} \quad \text{for } x \in \mathbb{R}.$ $e^{-x} \leq 1 - \frac{x}{2} \quad \text{for } x \in [0, 1.59]; \quad e^x \leq 1 + x + x^2 \quad \text{for } x < 1.79.$ $\left(1 + \frac{x}{p}\right)^p \geq \left(1 + \frac{x}{q}\right)^q \quad \text{for (i) } x > 0, p > q > 0,$ $\text{(ii) } -p < -q < x < 0, \text{ (iii) } -q > -p > x > 0. \text{ Reverse for:}$ $\text{(iv) } q < 0 < p, -q > x > 0, \text{ (v) } q < 0 < p, -p < x < 0.$		
logarithm	$\frac{x}{1+x} \leq \ln(1+x) \leq \frac{x(6+x)}{6+4x} \leq x \quad \text{for } x > -1.$ $\frac{2}{2+x} \leq \frac{1}{\sqrt{1+x+x^2/12}} \leq \frac{\ln(1+x)}{x} \leq \frac{1}{\sqrt{x+1}} \leq \frac{2+x}{2+2x} \quad \text{for } x > -1.$ $\ln(n) + \frac{1}{n+1} < \ln(n+1) < \ln(n) + \frac{1}{n} \leq \sum_{i=1}^n \frac{1}{i} \leq \ln(n) + 1 \quad \text{for } n \geq 1.$ $ \ln(x) \leq \frac{1}{2} x - \frac{1}{x} ; \quad \ln(x+y) \leq \ln(x) + \frac{y}{x}; \quad \ln(x) \leq y(x^{\frac{1}{y}} - 1); \quad x, y > 0.$ $\ln(1+x) \geq x - \frac{x^2}{2} \quad \text{for } x \geq 0; \quad \ln(1+x) \geq x - x^2 \quad \text{for } x \geq -0.68.$		
trigonometric	$x - \frac{x^3}{2} \leq x \cos x \leq \frac{x \cos x}{1-x^2/3} \leq x \sqrt[3]{\cos x} \leq x - x^3/6 \leq x \cos \frac{x}{\sqrt{3}} \leq \sin x,$		
hyperbolic	$x \cos x \leq \frac{x^3}{\sinh^2 x} \leq x \cos^2(x/2) \leq \sin x \leq (x \cos x + 2x)/3 \leq \frac{x^2}{\sinh x},$ $\max\left\{\frac{2}{\pi}, \frac{\pi^2 - x^2}{\pi^2 + x^2}\right\} \leq \frac{\sin x}{x} \leq \cos \frac{x}{2} \leq 1 \leq 1 + \frac{x^2}{3} \leq \frac{\tan x}{x} \quad \text{for } x \in [0, \frac{\pi}{2}].$		
square root	$2\sqrt{x+1} - 2\sqrt{x} < \frac{1}{\sqrt{x}} < \sqrt{x+1} - \sqrt{x-1} < 2\sqrt{x} - 2\sqrt{x-1} \quad \text{for } x \geq 1.$ $x \leq \frac{x+1}{2} - \frac{(x-1)^2}{2} \leq \sqrt{x} \leq \frac{x+1}{2} - \frac{(x-1)^2}{8} \quad \text{for } x \in [0, 1].$		
		<i>binomial</i>	
		$\max\left\{\frac{n^k}{k^k}, \frac{(n-k+1)^k}{k!}\right\} \leq \binom{n}{k} \leq \frac{n^k}{k!} \leq \left(\frac{en}{k}\right)^k; \quad \binom{n}{k} \leq \frac{n^n}{k^k(n-k)^{n-k}}.$ $\frac{n^k}{4k!} \leq \binom{n}{k} \quad \text{for } \sqrt{n} \geq k \geq 0; \quad \frac{4^n}{\sqrt{\pi n}} \left(1 - \frac{1}{8n}\right) \leq \binom{2n}{n} \leq \frac{4^n}{\sqrt{\pi n}} \left(1 - \frac{1}{9n}\right).$ $\binom{n_1}{k_1} \binom{n_2}{k_2} \leq \binom{n_1+n_2}{k_1+k_2}; \quad \binom{tn}{k} \geq t^k \binom{n}{k} \quad \text{for } t \geq 1.$ $\frac{\sqrt{\pi}}{2} G \leq \binom{n}{\alpha n} \leq G \quad \text{for } G = \frac{2^{nH(\alpha)}}{\sqrt{2\pi n\alpha(1-\alpha)}}, \quad H(x) = -\log_2(x^x(1-x)^{1-x}).$ $\sum_{i=0}^d \binom{n}{i} \leq \min\left\{n^d + 1, \left(\frac{en}{d}\right)^d, 2^n\right\} \quad \text{for } n \geq d \geq 1.$ $\sum_{i=0}^{\alpha n} \binom{n}{i} \leq \min\left\{\frac{1-\alpha}{1-2\alpha} \binom{n}{\alpha n}, 2^{nH(\alpha)}, 2^n e^{-2n(\frac{1}{2}-\alpha)^2}\right\} \quad \text{for } \alpha \in (0, \frac{1}{2}).$	
		<i>binary entropy</i>	
		$4x(1-x) \leq H(x) \leq (4x(1-x))^{1/\ln(4)} \quad \text{for } x \in (0, 1).$	
		<i>Stirling</i>	
		$e\left(\frac{n}{e}\right)^n \leq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{1/(12n+1)} \leq n! \leq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{1/12n} \leq en\left(\frac{n}{e}\right)^n$	
		<i>means</i>	
		$\min x_i \leq \frac{n}{\sum x_i} \leq (\prod x_i)^{1/n} \leq \frac{1}{n} \sum x_i \leq \sqrt{\frac{1}{n} \sum x_i^2} \leq \frac{\sum x_i^2}{\sum x_i} \leq \max x_i$	
		<i>power means</i>	
		$M_p \leq M_q \quad \text{for } p \leq q, \text{ where } M_p = (\sum_i w_i x_i ^p)^{1/p}, \quad w_i \geq 0, \sum_i w_i = 1.$	
		$\text{In the limit } M_0 = \prod_i x_i ^{w_i}, \quad M_{-\infty} = \min_i \{x_i\}, \quad M_\infty = \max_i \{x_i\}.$	
		<i>Lehmer</i>	
		$\frac{\sum_i w_i x_i ^p}{\sum_i w_i x_i ^{p-1}} \leq \frac{\sum_i w_i x_i ^q}{\sum_i w_i x_i ^{q-1}} \quad \text{for } p \leq q, \quad w_i \geq 0.$	
		<i>log mean</i>	
		$\sqrt{xy} \leq \left(\frac{\sqrt{x}+\sqrt{y}}{2}\right) (xy)^{\frac{1}{4}} \leq \frac{x-y}{\ln(x)-\ln(y)} \leq \left(\frac{\sqrt{x}+\sqrt{y}}{2}\right)^2 \leq \frac{x+y}{2} \quad \text{for } x, y > 0.$	
		<i>Heinz</i>	
		$\sqrt{xy} \leq \frac{x^{1-\alpha} y^\alpha + x^\alpha y^{1-\alpha}}{2} \leq \frac{x+y}{2} \quad \text{for } x, y > 0, \quad \alpha \in [0, 1].$	
		<i>Maclaurin-Newton</i>	
		$S_k^2 \geq S_{k-1} S_{k+1} \quad \text{and} \quad (S_k)^{1/k} \geq (S_{k+1})^{1/(k+1)} \quad \text{for } 1 \leq k < n,$ $S_k = \frac{1}{\binom{n}{k}} \sum_{1 \leq i_1 < \dots < i_k \leq n} a_{i_1} a_{i_2} \dots a_{i_k}, \quad \text{and} \quad a_i \geq 0.$	
		<i>Jensen</i>	
		$\varphi(\sum_i p_i x_i) \leq \sum_i p_i \varphi(x_i) \quad \text{where } p_i \geq 0, \sum p_i = 1, \text{ and } \varphi \text{ convex.}$	
		$\text{Alternatively: } \varphi(\mathbf{E}[X]) \leq \mathbf{E}[\varphi(X)]. \text{ For concave } \varphi \text{ the reverse holds.}$	
		<i>Chebyshev</i>	
		$\sum_{i=1}^n f(x_i)g(x_i)p_i \geq \left(\sum_{i=1}^n f(x_i)p_i\right) \left(\sum_{i=1}^n g(x_i)p_i\right)$	
		$\text{for } x_1 \leq \dots \leq x_n \text{ and } f, g \text{ nondecreasing, } p_i \geq 0, \quad \sum p_i = 1.$	
		$\text{Alternatively: } \mathbf{E}[f(X)g(X)] \geq \mathbf{E}[f(X)]\mathbf{E}[g(X)].$	
		<i>rearrangement</i>	
		$\sum_{i=1}^n a_i b_i \geq \sum_{i=1}^n a_i b_{\pi(i)} \geq \sum_{i=1}^n a_i b_{n-i+1} \quad \text{for } a_1 \leq \dots \leq a_n,$ $b_1 \leq \dots \leq b_n \text{ and } \pi \text{ a permutation of } [n]. \text{ More generally:}$ $\sum_{i=1}^n f_i(b_i) \geq \sum_{i=1}^n f_i(b_{\pi(i)}) \geq \sum_{i=1}^n f_i(b_{n-i+1})$	
		$\text{with } (f_{i+1}(x) - f_i(x)) \text{ nondecreasing for all } 1 \leq i < n.$	
		$\text{Dually: } \prod_{i=1}^n (a_i + b_i) \leq \prod_{i=1}^n (a_i + b_{\pi(i)}) \leq \prod_{i=1}^n (a_i + b_{n-i+1}) \quad \text{for } a_i, b_i \geq 0.$	

Weierstrass	$1 - \sum_i w_i x_i \leq \prod_i (1 - x_i)^{w_i}, \quad \text{and}$ $1 + \sum_i w_i x_i \leq \prod_i (1 + x_i)^{w_i} \leq \prod_i (1 - x_i)^{-w_i} \quad \text{for } x_i \in [0, 1], w_i \geq 1.$ $\prod_i (1 \pm x_i)^{w_i} \leq (1 \mp \sum_i w_i x_i)^{-1} \quad \text{if additionally } \sum_i w_i x_i \leq 1.$	Milne	$(\sum_{i=1}^n (a_i + b_i)) \left(\sum_{i=1}^n \frac{a_i b_i}{a_i + b_i} \right) \leq (\sum_{i=1}^n a_i) (\sum_{i=1}^n b_i) \quad \text{for } a_i, b_i \geq 0.$
Kantorovich	$(\sum_i x_i^2) (\sum_i y_i^2) \leq \left(\frac{A}{G}\right)^2 (\sum_i x_i y_i)^2 \quad \text{for } x_i, y_i > 0,$ $0 < m \leq \frac{x_i}{y_i} \leq M < \infty, \quad A = (m + M)/2, \quad G = \sqrt{mM}.$	Carleman	$\sum_{k=1}^n \left(\prod_{i=1}^k a_i \right)^{1/k} \leq e \sum_{k=1}^n a_k $
Nesbitt	$\sum_i^n \frac{a_i}{S-a_i} \geq \frac{n}{n-1} \quad \text{for } a_i \geq 0, \quad S = \sum_{i=1}^n a_i.$	sum & product	$ \prod_{i=1}^n a_i - \prod_{i=1}^n b_i \leq \sum_{i=1}^n a_i - b_i \quad \text{for } a_i , b_i \leq 1.$ $\prod_{i=1}^n (t + a_i) \geq (t + 1)^n \quad \text{where } \prod_{i=1}^n a_i \geq 1, \quad a_i > 0, \quad t > 0.$
Cauchy	$\sum_{L-1}^U f(x) dx \leq \sum_{i=L}^U f(i) \leq \int_L^{U+1} f(x) dx \quad \text{for } f \text{ nondecreasing.}$	Radon	$\sum_i \frac{x_i^p}{a_i^{p-1}} \geq \frac{(\sum_i x_i)^p}{(\sum_i a_i)^{p-1}} \quad \text{for } x_i, a_i \geq 0, \quad p \geq 1 \text{ (rev. if } p \in [0, 1]).$
Hermite	$\varphi\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b \varphi(x) dx \leq \frac{\varphi(a)+\varphi(b)}{2} \quad \text{for } \varphi \text{ convex.}$	Karamata	$\sum_{i=1}^n \varphi(a_i) \geq \sum_{i=1}^n \varphi(b_i) \quad \text{for } a_1 \geq a_2 \geq \dots \geq a_n, \quad b_1 \geq \dots \geq b_n,$ and $\{a_i\} \succeq \{b_i\}$ (majorization), i.e. $\sum_{i=1}^t a_i \geq \sum_{i=1}^t b_i$ for all $1 \leq t \leq n$, with $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$, and φ convex (for concave φ the reverse holds).
Gibbs	$\sum_i a_i \log \frac{a_i}{b_i} \geq a \log \frac{a}{b} \quad \text{for } a_i, b_i \geq 0, \text{ or more generally:}$ $\sum_i a_i \varphi\left(\frac{b_i}{a_i}\right) \leq a \varphi\left(\frac{b}{a}\right) \quad \text{for } \varphi \text{ concave, and } a = \sum a_i, \quad b = \sum b_i.$	Muirhead	$\sum_{\pi} x_{\pi(1)}^{a_1} \cdots x_{\pi(n)}^{a_n} \geq \sum_{\pi} x_{\pi(1)}^{b_1} \cdots x_{\pi(n)}^{b_n}, \quad \text{sums over permut. } \pi \text{ of } [n],$ where $a_1 \geq \dots \geq a_n, \quad b_1 \geq \dots \geq b_n, \quad \{a_k\} \succeq \{b_k\}, \quad x_i \geq 0.$
Chong	$\sum_{i=1}^n \frac{a_i}{a_{\pi(i)}} \geq n \quad \text{and} \quad \prod_{i=1}^n a_i^{a_i} \geq \prod_{i=1}^n a_i^{a_{\pi(i)}} \quad \text{for } a_i > 0.$	Hilbert	$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_m b_n}{m+n} \leq \pi \left(\sum_{m=1}^{\infty} a_m^2 \right)^{\frac{1}{2}} \left(\sum_{n=1}^{\infty} b_n^2 \right)^{\frac{1}{2}} \quad \text{for } a_m, b_n \in \mathbb{R}.$
Schur-Vornicu	$f(x)(x-y)^k(x-z)^k + f(y)(y-z)^k(y-x)^k + f(z)(z-x)^k(z-y)^k \geq 0$ where $x, y, z \geq 0$, $k \geq 1$ integer, f convex or monotonic, $f \geq 0$.	Hardy	With $\max\{m, n\}$ instead of $m + n$, we have 4 instead of π .
Young	$(\frac{1}{px^p} + \frac{1}{qy^q})^{-1} \leq xy \leq \frac{x^p}{p} + \frac{y^q}{q} \quad \text{for } x, y, p, q > 0, \quad \frac{1}{p} + \frac{1}{q} = 1.$ $\int_0^a f(x) dx + \int_0^b f^{-1}(x) dx \geq ab, \text{ for } f \text{ cont., strictly increasing.}$	Mathieu	$\sum_{n=1}^{\infty} \left(\frac{a_1+a_2+\dots+a_n}{n} \right)^p \leq \left(\frac{p}{p-1} \right)^p \sum_{n=1}^{\infty} a_n^p \quad \text{for } a_n \geq 0, p > 1.$
Shapiro	$\sum_{i=1}^n \frac{x_i}{x_{i+1}+x_{i+2}} \geq \frac{n}{2} \quad \text{where } x_i > 0, \quad (x_{n+1}, x_{n+2}) := (x_1, x_2),$ and $n \leq 12$ if even, $n \leq 23$ if odd.	Kraft	$\sum 2^{-c(i)} \leq 1 \quad \text{for } c(i) \text{ depth of leaf } i \text{ of binary tree, sum over all leaves.}$
Hadamard	$(\det A)^2 \leq \prod_{i=1}^n \sum_{j=1}^n A_{ij}^2 \quad \text{where } A \text{ is an } n \times n \text{ matrix.}$	LYM	$\sum_{X \in \mathcal{A}} \binom{n}{ X }^{-1} \leq 1, \quad \mathcal{A} \subset 2^{[n]}, \text{ no set in } \mathcal{A} \text{ is subset of another set in } \mathcal{A}.$
Schur	$\sum_{i=1}^n \lambda_i^2 \leq \sum_{i=1}^n \sum_{j=1}^n A_{ij}^2 \quad \text{and} \quad \sum_{i=1}^k d_i \leq \sum_{i=1}^k \lambda_i \quad \text{for } 1 \leq k \leq n.$ A is an $n \times n$ matrix. For the second inequality A is symmetric. $\lambda_1 \geq \dots \geq \lambda_n$ the eigenvalues, $d_1 \geq \dots \geq d_n$ the diagonal elements.	FKG	$\Pr[x \in \mathcal{A} \cap \mathcal{B}] \geq \Pr[x \in \mathcal{A}] \cdot \Pr[x \in \mathcal{B}], \quad \text{for } \mathcal{A}, \mathcal{B} \text{ monotone set systems.}$
Ky Fan	$\frac{\prod_{i=1}^n x_i^{a_i}}{\prod_{i=1}^n (1 - x_i)^{a_i}} \leq \frac{\sum_{i=1}^n a_i x_i}{\sum_{i=1}^n a_i (1 - x_i)} \quad \text{for } x_i \in [0, \frac{1}{2}], a_i \in [0, 1], \sum a_i = 1.$	Shearer	$ \mathcal{A} ^t \leq \prod_{F \in \mathcal{F}} \text{trace}_F(\mathcal{A}) \quad \text{for } \mathcal{A}, \mathcal{F} \subseteq 2^{[n]}, \text{ where every } i \in [n]$ appears in at least t sets of \mathcal{F} , and $\text{trace}_F(\mathcal{A}) = \{F \cap A : A \in \mathcal{A}\}$.
Aczél	$(a_1 b_1 - \sum_{i=2}^n a_i b_i)^2 \geq (a_1^2 - \sum_{i=2}^n a_i^2)(b_1^2 - \sum_{i=2}^n b_i^2)$ given that $a_1^2 > \sum_{i=2}^n a_i^2$ or $b_1^2 > \sum_{i=2}^n b_i^2$.	Sauer-Shelah	$ \mathcal{A} \leq \text{str}(\mathcal{A}) \leq \sum_{i=0}^{\text{vc}(\mathcal{A})} \binom{n}{i} \leq n^{\text{vc}(\mathcal{A})} + 1 \quad \text{for } \mathcal{A} \subseteq 2^{[n]}, \text{ and}$ $\text{str}(\mathcal{A}) = \{X \subseteq [n] : \text{trace}_X(\mathcal{A}) = 2^X\}, \quad \text{vc}(\mathcal{A}) = \max\{ X : X \in \text{str}(\mathcal{A})\}.$
Mahler	$\prod_{i=1}^n (x_i + y_i)^{1/n} \geq \prod_{i=1}^n x_i^{1/n} + \prod_{i=1}^n y_i^{1/n} \quad \text{where } x_i, y_i > 0.$	Khintchine	$\sqrt{\sum_i a_i^2} \geq \mathbb{E}[\sum_i a_i r_i] \geq \frac{1}{\sqrt{2}} \sqrt{\sum_i a_i^2} \quad \text{where } a_i \in \mathbb{R}, \text{ and}$ $r_i \in \{\pm 1\}$ random variables (r.v.) i.i.d. w.pr. $\frac{1}{2}$.
Abel	$b_1 \cdot \min_k \sum_{i=1}^k a_i \leq \sum_{i=1}^n a_i b_i \leq b_1 \cdot \max_k \sum_{i=1}^k a_i \quad \text{for } b_1 \geq \dots \geq b_n \geq 0.$	Bonferroni	$\Pr\left[\bigvee_{i=1}^n A_i\right] \leq \sum_{j=1}^k (-1)^{j-1} S_j \quad \text{for } 1 \leq k \leq n, \quad k \text{ odd (rev. for } k \text{ even),}$ $S_k = \sum_{1 \leq i_1 < \dots < i_k \leq n} \Pr[A_{i_1} \wedge \dots \wedge A_{i_k}] \quad \text{where } A_i \text{ are events.}$
		Bhatia-Davis	$\text{Var}[X] \leq (M - \mathbb{E}[X])(\mathbb{E}[X] - m) \quad \text{where } X \in [m, M].$

Samuelson	$\mu - \sigma\sqrt{n-1} \leq x_i \leq \mu + \sigma\sqrt{n-1}$ for $i = 1, \dots, n$, where $\mu = \sum x_i/n$, $\sigma^2 = \sum (x_i - \mu)^2/n$.	Paley-Zygmund	$\Pr[X \geq \mu \text{ E}[X]] \geq 1 - \frac{\text{Var}[X]}{(1-\mu)^2 (\text{E}[X])^2 + \text{Var}[X]}$ for $X \geq 0$, $\text{Var}[X] < \infty$, and $\mu \in (0, 1)$.
Markov	$\Pr[X \geq a] \leq \text{E}[X]/a$ where X is a r.v., $a > 0$. $\Pr[X \leq c] \leq (1 - \text{E}[X])/(1 - c)$ for $X \in [0, 1]$ and $c \in [0, \text{E}[X]]$. $\Pr[X \in S] \leq \text{E}[f(X)]/s$ for $f \geq 0$, and $f(x) \geq s > 0$ for all $x \in S$.	Vysotskij	$\Pr[X - \text{E}[X] \geq \lambda\sigma] \leq \frac{4}{9\lambda^2}$ if $\lambda \geq \sqrt{\frac{8}{3}}$,
Chebyshev	$\Pr[X - \text{E}[X] \geq t] \leq \text{Var}[X]/t^2$ where $t > 0$. $\Pr[X - \text{E}[X] \geq t] \leq \text{Var}[X]/(\text{Var}[X] + t^2)$ where $t > 0$.	Petunin-Gauss	$\Pr[X - m \geq \varepsilon] \leq \frac{4\tau^2}{9\varepsilon^2}$ if $\varepsilon \geq \frac{2\tau}{\sqrt{3}}$, $\Pr[X - m \geq \varepsilon] \leq 1 - \frac{\varepsilon}{\sqrt{3}\tau}$ if $\varepsilon \leq \frac{2\tau}{\sqrt{3}}$.
2nd moment	$\Pr[X > 0] \geq (\text{E}[X])^2/(\text{E}[X^2])$ where $\text{E}[X] \geq 0$. $\Pr[X = 0] \leq \text{Var}[X]/(\text{E}[X^2])$ where $\text{E}[X^2] \neq 0$.	Etemadi	Where X is a unimodal r.v. with mode m , $\sigma^2 = \text{Var}[X] < \infty$, $\tau^2 = \text{Var}[X] + (\text{E}[X] - m)^2 = \text{E}[(X - m)^2]$.
kth moment	$\Pr[X - \mu \geq t] \leq \frac{\text{E}[(X - \mu)^k]}{t^k}$ and $\Pr[X - \mu \geq t] \leq C_k \left(\frac{nk}{et^2} \right)^{k/2}$ for $X_i \in [0, 1]$ k-wise indep. r.v., $X = \sum X_i$, $i = 1, \dots, n$, $\mu = \text{E}[X]$, $C_k = 2\sqrt{\pi k}e^{1/6k}$, k even.	Doob	$\Pr[\max_{1 \leq k \leq n} S_k \geq 3\alpha] \leq 3 \max_{1 \leq k \leq n} (\Pr[S_k \geq \alpha])$ where X_i are i.r.v., $S_k = \sum_{i=1}^k X_i$, $\alpha \geq 0$.
4th moment	$\text{E}[X] \geq \frac{(\text{E}[X^2])^{3/2}}{(\text{E}[X^4])^{1/2}}$ where $0 < \text{E}[X^4] < \infty$.	Bennett	$\Pr[\sum_{i=1}^n X_i \geq \varepsilon] \leq \exp\left(-\frac{n\sigma^2}{M^2} \theta\left(\frac{M\varepsilon}{n\sigma^2}\right)\right)$ where X_i i.r.v., $\text{E}[X_i] = 0$, $\sigma^2 = \frac{1}{n} \sum \text{Var}[X_i]$, $ X_i \leq M$ (w. prob. 1), $\varepsilon \geq 0$, $\theta(u) = (1+u)\log(1+u) - u$.
Chernoff	$\Pr[X \geq t] \leq F(a)/a^t$ for X r.v., $\Pr[X = k] = p_k$, $F(z) = \sum_k p_k z^k$ probability gen. func., and $a \geq 1$. $\Pr[X \geq \Delta] = \Pr[X \leq -\Delta] \leq \exp(-\Delta^2/2n)$ for $X_i \in \{0, 1\}$ i.r.v., and $\Pr[X_i = \pm 1] = \frac{1}{2}$ for $i = 1, \dots, n$, $X = \sum X_i$, $\Delta \geq 0$. $\Pr[X \geq (1+\delta)\mu] \leq \left(\frac{e^\delta}{(1+\delta)^{(1+\delta)}}\right)^\mu \leq \exp\left(\frac{-\mu\delta^2}{2+\delta}\right)$ $\Pr[X \leq (1-\delta)\mu] \leq \left(\frac{e^{-\delta}}{(1-\delta)^{(1-\delta)}}\right)^\mu \leq \exp\left(\frac{-\mu\delta^2}{2}\right)$ for $X_i \in [0, 1]$ i.r.v., $X = \sum X_i$, $\mu = \text{E}[X]$, $\delta \geq 0$. Further from the mean: $\Pr[X \geq R] \leq 2^{-R}$ for $R \geq 2e\mu$ ($\approx 5.44\mu$). $\Pr[\sum X_i \geq t] \leq \binom{n}{k} p^k / \binom{t}{k}$ for $X_i \in \{0, 1\}$ k-wise i.r.v., $\text{E}[X_i] = p$. $\Pr[X \geq (1+\delta)\mu] \leq \binom{n}{k} p^k / \binom{(1+\delta)\mu}{k}$ for $X_i \in [0, 1]$ k-wise i.r.v., $k \geq \hat{k} = \lceil \mu\delta/(1-p) \rceil$, $\text{E}[X_i] = p_i$, $X = \sum X_i$, $\mu = \text{E}[X]$, $p = \frac{\mu}{n}$, $\delta > 0$.	Bernstein	$\Pr[\sum_{i=1}^n X_i \geq \varepsilon] \leq \exp\left(\frac{-\varepsilon^2}{2(n\sigma^2 + M\varepsilon/3)}\right)$ for X_i i.r.v., $\text{E}[X_i] = 0$, $ X_i < M$ (w. prob. 1) for all i , $\sigma^2 = \frac{1}{n} \sum \text{Var}[X_i]$, $\varepsilon \geq 0$.
Hoeffding	$\Pr[X - \text{E}[X] \geq \delta] \leq 2 \exp\left(\frac{-2\delta^2}{\sum_{i=1}^n (b_i - a_i)^2}\right)$ for X_i i.r.v., $X_i \in [a_i, b_i]$ (w. prob. 1), $X = \sum X_i$, $\delta \geq 0$. A related lemma, assuming $\text{E}[X] = 0$, $X \in [a, b]$ (w. prob. 1) and $\lambda \in \mathbb{R}$: $\text{E}[e^{\lambda X}] \leq \exp\left(\frac{\lambda^2(b-a)^2}{8}\right)$	Azuma	$\Pr[X_n - X_0 \geq \delta] \leq 2 \exp\left(\frac{-\delta^2}{2 \sum_{i=1}^n c_i^2}\right)$ for martingale (X_k) s.t. $ X_i - X_{i-1} < c_i$ (w. prob. 1), for $i = 1, \dots, n$, $\delta \geq 0$.
Kolmogorov	$\Pr[\max_k S_k \geq \varepsilon] \leq \frac{1}{\varepsilon^2} \text{Var}[S_n] = \frac{1}{\varepsilon^2} \sum_i \text{Var}[X_i]$ where X_1, \dots, X_n are i.r.v., $\text{E}[X_i] = 0$, $\text{Var}[X_i] < \infty$ for all i , $S_k = \sum_{i=1}^k X_i$ and $\varepsilon > 0$.	Efron-Stein	$\text{Var}[Z] \leq \frac{1}{2} \text{E}\left[\sum_{i=1}^n (Z - Z^{(i)})^2\right]$ for $X_i, X_i' \in \mathcal{X}$ i.r.v., $f : \mathcal{X}^n \rightarrow \mathbb{R}$, $Z = f(X_1, \dots, X_n)$, $Z^{(i)} = f(X_1, \dots, X_{i-1}, X_i, \dots, X_n)$.
		McDiarmid	$\Pr[Z - \text{E}[Z] \geq \delta] \leq 2 \exp\left(\frac{-2\delta^2}{\sum_{i=1}^n c_i^2}\right)$ for $X_i, X_i' \in \mathcal{X}$ i.r.v., $Z, Z^{(i)}$ as before, s.t. $ Z - Z^{(i)} \leq c_i$ for all i , and $\delta \geq 0$.
		Janson	$M \leq \Pr[\bigwedge \bar{B}_i] \leq M \exp\left(\frac{\Delta}{2-2\varepsilon}\right)$ where $\Pr[B_i] \leq \varepsilon$ for all i , $M = \prod_i (1 - \Pr[B_i])$, $\Delta = \sum_{i \neq j, B_i \sim B_j} \Pr[B_i \wedge B_j]$.
		Lovász	$\Pr[\bigwedge \bar{B}_i] \geq \prod_i (1 - x_i) > 0$ where $\Pr[B_i] \leq x_i \cdot \prod_{(i,j) \in D} (1 - x_j)$, for $x_i \in [0, 1]$ for all $i = 1, \dots, n$ and D the dependency graph. If each B_i mutually indep. of all other events, except at most d , $\Pr[B_i] \leq p$ for all $i = 1, \dots, n$, then if $ep(d+1) \leq 1$ then $\Pr[\bigwedge \bar{B}_i] > 0$.